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Vibration Tests of a Steel-Frame Building

by

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Summary

The natural frequency and damping of a steel-frame building were experimentally determined for the first two modes of vibration from a steady state resonance curve obtained with a variable frequency mechanical oscillator. The value of the first natural frequency was 3.5 cps, which may be compared with the 2.9 cps frequency which was determined from calculations of the response to the transient excitation of a quarry blast. The difference in these frequencies is explained by the fact that somewhat different modes of vibration were excited by the two tests, and by possible differences in structural damping at the high and low amplitude levels of the two tests. The maximum accelerations set up during the steady state tests were of the order of 0.005 g., whereas the accelerations during the blast reached values around 0.1 g. The damping measured for the low amplitude steady state vibrations was 3.4% of critical damping.

Introduction

In a previous report [1] the results of measurements of accelerations in a steel-frame building caused by a quarry blast were given. From the ground acceleration - time record produced by the blast, an acceleration response spectrum curve was computed, from which the building accelerations could be determined if the natural frequency and damping of the structure were known. Since the building accelerations during the blast were measured as well as the ground accelerations, it was possible to work backward from the response spectrum and to calculate the effective vibration frequencies and damping characteristics which must have been involved.

The object of the present investigation was to obtain an independent determination of the natural frequencies and damping of the building. The method was to subject the structure to a sinusoidally varying force and to determine the steady-state acceleration response of the building at various frequencies. From this experimentally determined resonance curve the natural frequencies and damping were determined and were compared with the values obtained from the transient measurements made during the blast.

The Structure

A general external view of the test building is shown in Fig. 1, and Fig. 2 indicates the type of construction used. The overall dimensions and structural member sizes are given in Figures 3 and 4.

For purposes of the present test the multi-story end of the building may be considered as a separate unit. The structure can be idealized as a two-degree-of-freedom system involving a translational and a torsional mode of the heavy floor slab located at about the mid-height of the building, as shown in Fig. 5.

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1) Hudson, D. E., Alford, J. L., and Housner, G. W., Response of a Structure to an Explosive-Generated Ground Shock, Third Technical Report, ONR-081-093, California Institute of Technology, Pasadena, California, September 1952.

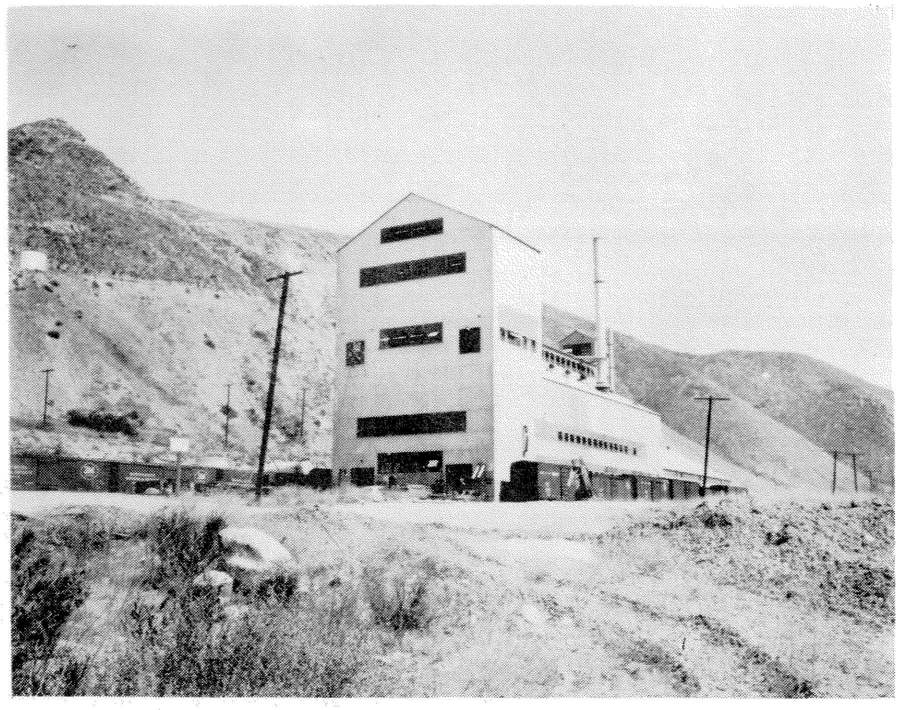

FIGURE 1. General View of Test Building

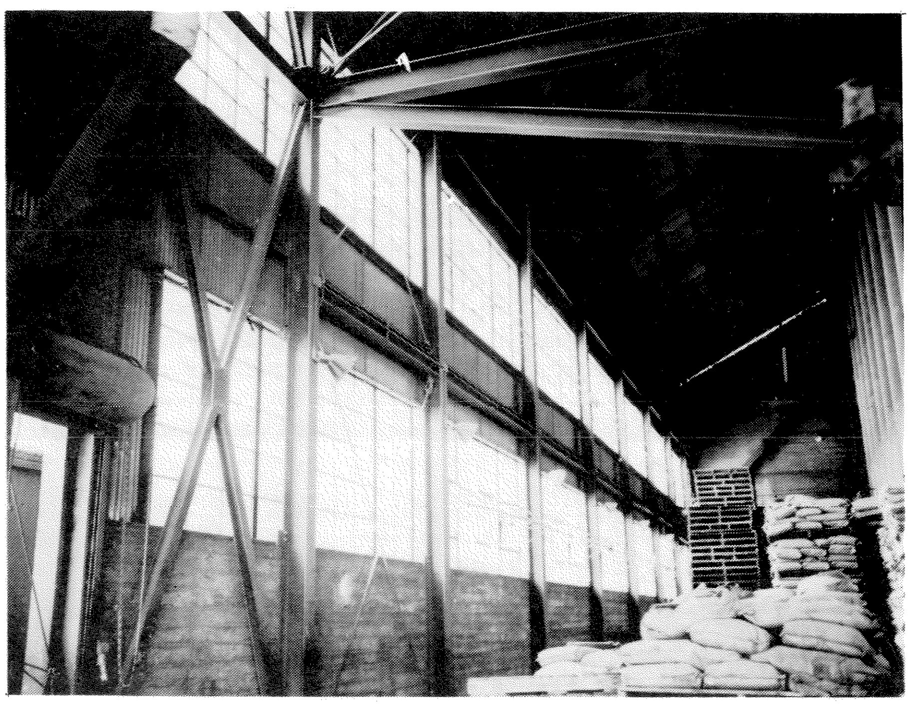

FIGURE 2. Construction Details of Test Building

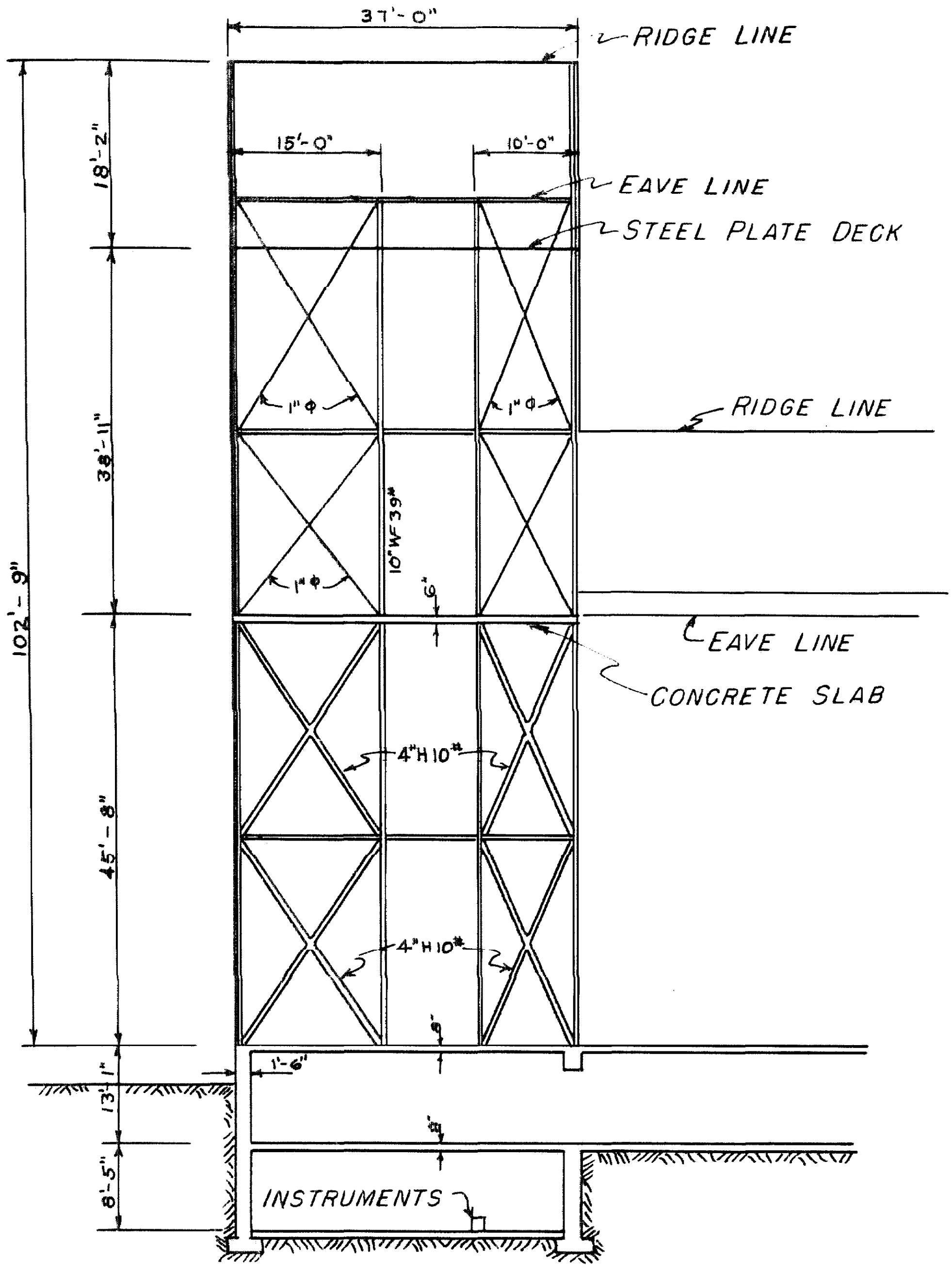

FIG. 3. DIMENSIONAL DETAILS OF TEST BUILDING

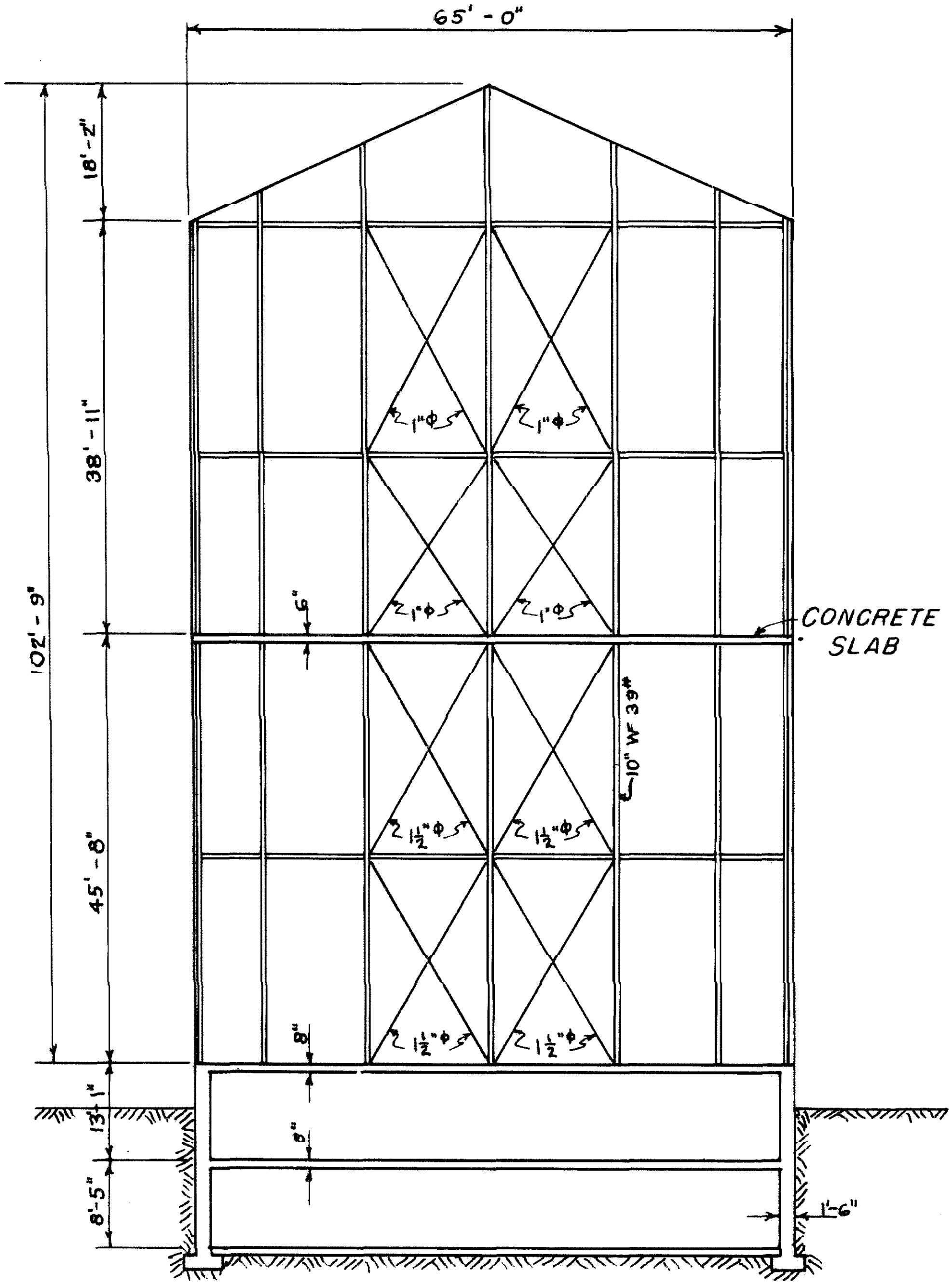

FIG. 4. DIMENSIONAL DETAILS OF TEST BUILDING

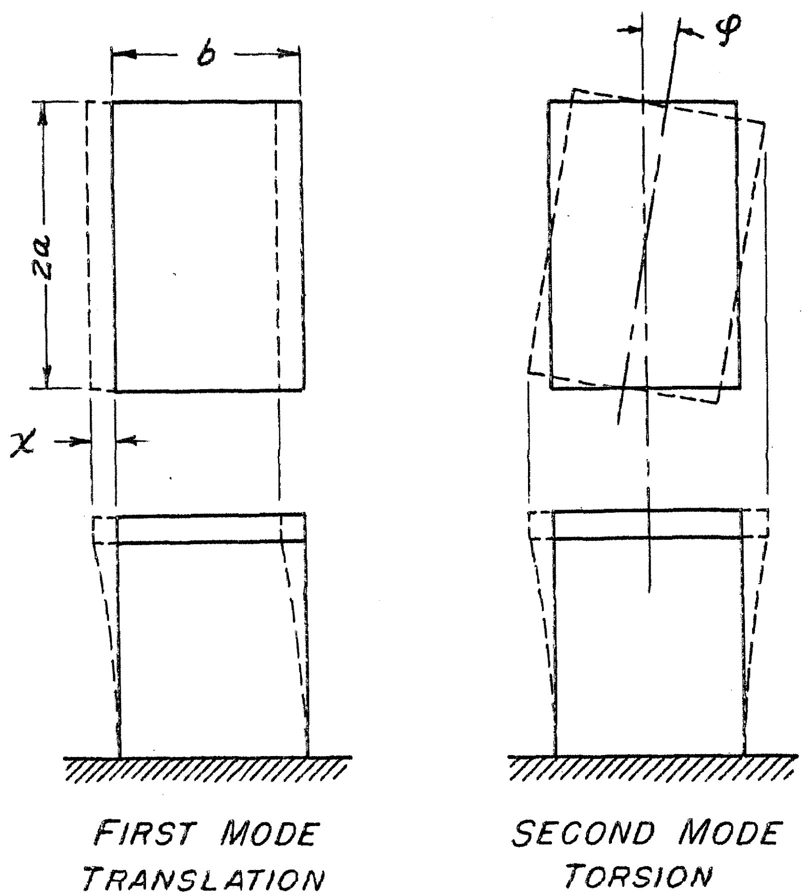

FIG. 5. VIBRATION MODES
OF SIMPLIFIED STRUCTURE

The Vibration Exciter

The vibration exciter was a rotating unbalanced-weight type borrowed from the United States Coast and Geodetic Survey. The unit was modified for the present test by adding a support bracket assembly by means of which the whole exciter unit could be bolted to the floor, and by the addition of a thyratron controlled variable speed D. C. motor drive system which permitted a close adjustment of speed. The exciter consisted of two counter-rotating eccentric weights, each having a static unbalance of 59.3 lb in., so arranged in phase that the resultant inertia force was horizontal. As may be seen in Fig. 6 the two exciter weights were carried on concentric wheels, one of which rotated within the other, so coupled together by gears that the rotational velocities were equal in magnitude and opposite in direction. The total resultant horizontal inertia force produced by the unit, for the frequency range used in the present tests, may be seen in Fig. 7.

The Response Measurements

The building response accelerations were measured by two accelerometers fastened to the upper floor slab. The location of these pickups are shown in Fig. 8, which also indicates the exciter location. A sufficient distance between pickups was provided so that the torsional mode could be distinguished from the translational mode. The pickup and recording system, Fig. 9, was the same one used in the blast investigation. The accelerometer was a small seismic type instrument having a natural frequency of about 80 cycles per second and about 60% of critical fluid damping, and a measuring element of the variable reluctance balanced armature type. The signal from the accelerometer was amplified by a carrier-type amplifying system, and was recorded on a direct inking oscillograph. The amplifier-oscillograph combination had an overall frequency response which was essentially constant from 0 - 100 cycles per second.

Overall acceleration amplitude calibrations were obtained by rotating the accelerometers through 90° in the earth's gravitational field,

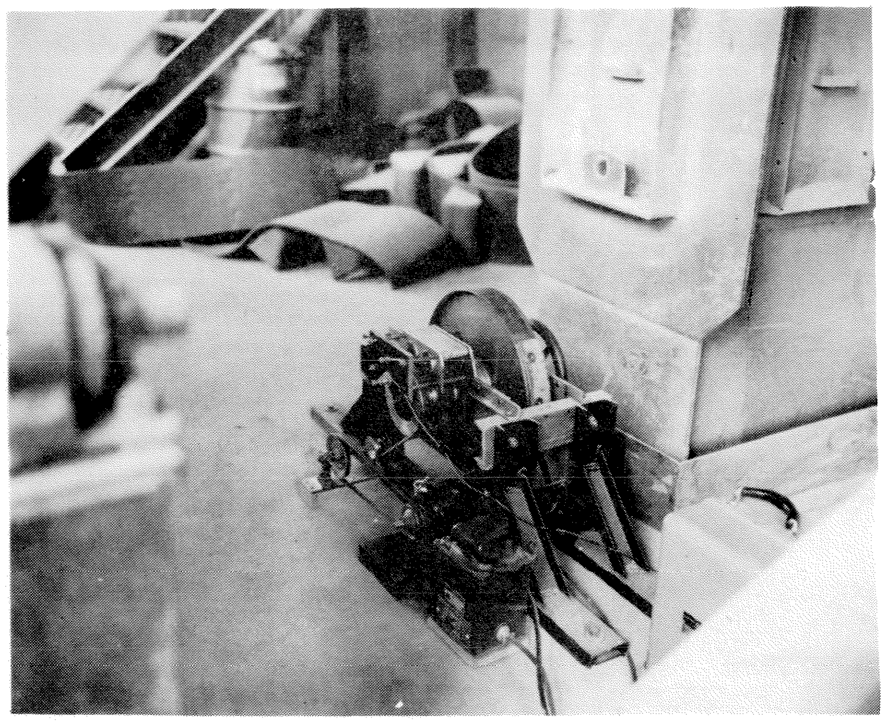

FIGURE 6. Vibration Exciter and Drive Unit

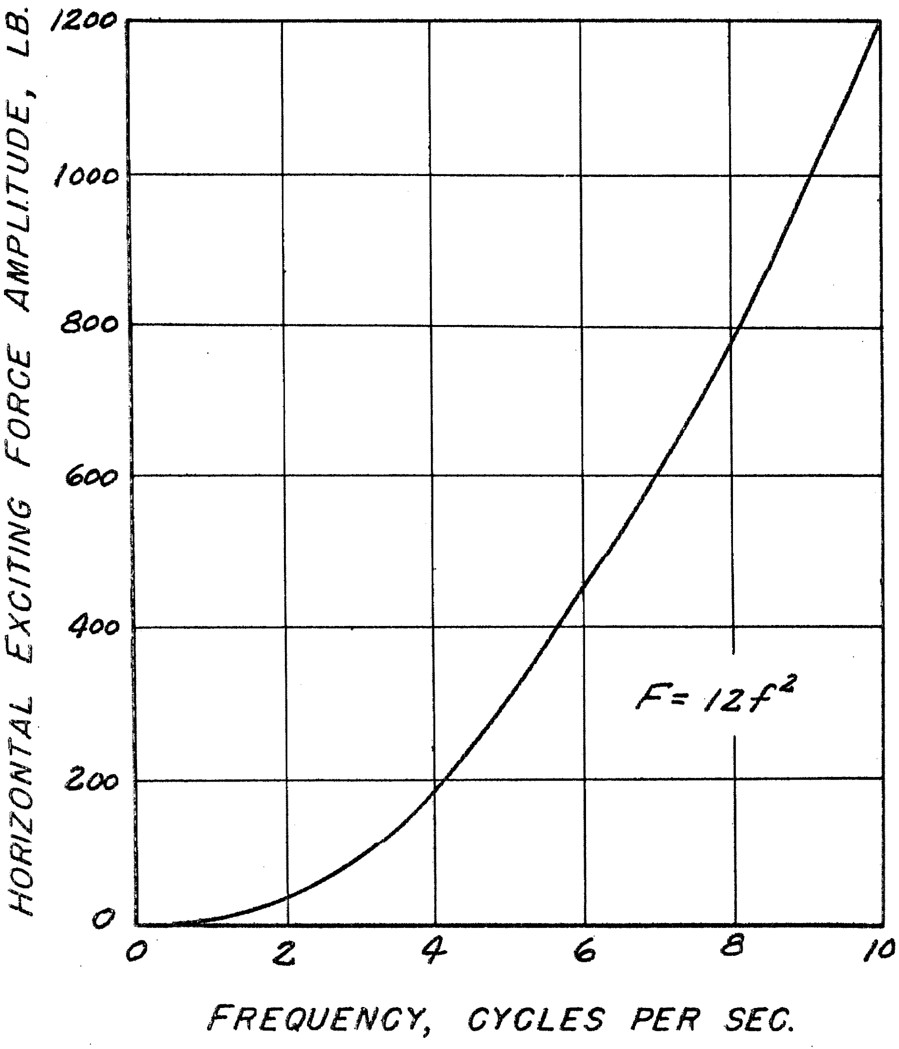

FIG. 7. EXCITER OUTPUT FORCE

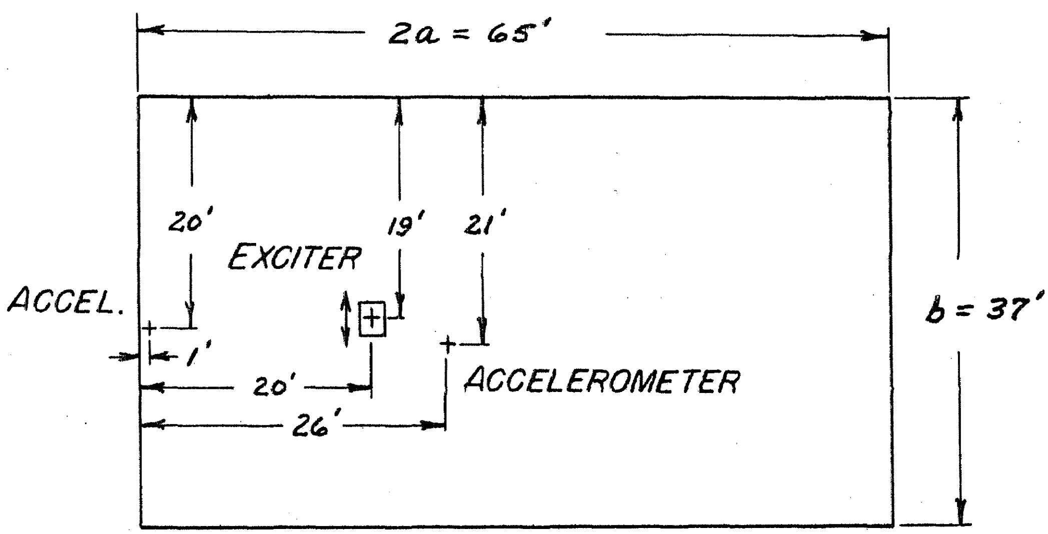

FIG. 8. LOCATION OF EXCITER UNITS AND ACCELEROMETERS

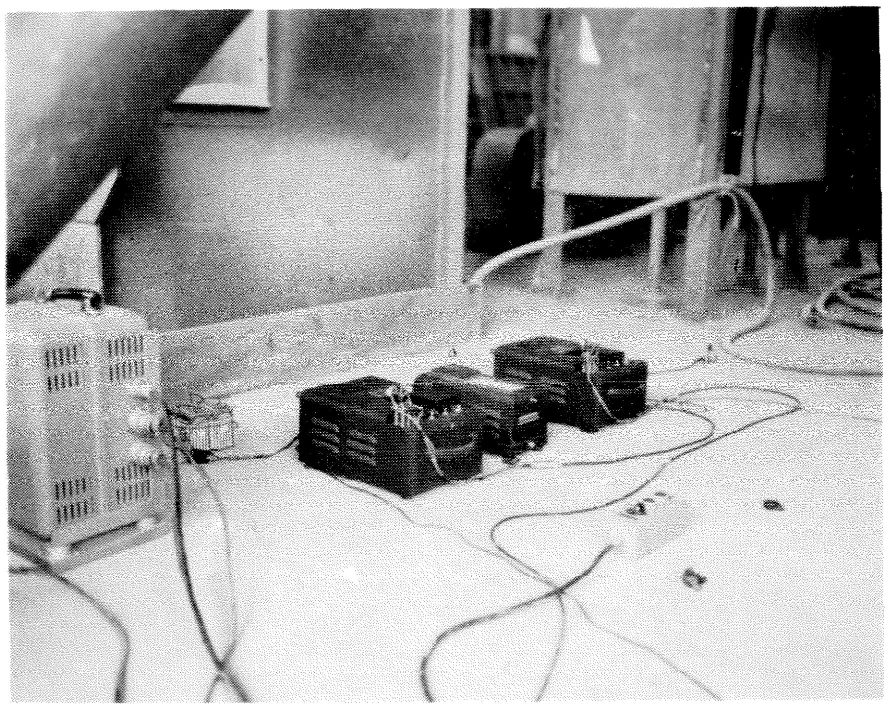

FIGURE 9. Instrument and Recording Installation

thus producing a D. C. signal equivalent to 1g acceleration. Paper speed calibrations were obtained by placing timed marks on the recording paper before and after the tests. Laboratory paper speed measurements were also made which showed that the speed remained constant over a wide range of instrument supply voltages.

Test Results

The only difficulties encountered during the tests arose from the fact that, at the low frequencies involved, only small exciting forces could be generated, and hence small building accelerations had to be measured. It was not possible to shut down all of the machinery operating in the building, and the relatively high frequency vibrations transmitted from some of this machinery interfered in some cases with the measurements of the basic building vibration. However, in spite of this low exciting force, usable records were obtained, and values of natural frequency were determined which are believed to be accurate to within ± 5%.

The amplitudes of motion excited in the first translational mode were so small that no resonance curve could be plotted. The resonant frequency could, however, be approximately determined, since only near one frequency could a clearly defined sinusoidal oscillation be recognized. This frequency could be readily checked by comparing it with the frequency of the second, or torsional, mode of vibration, since there is a simple relation between them which can be shown as follows.

If M = mass of floor slab,

I = polar moment of inertia of floor slab about vertical axis through center of mass,

K = equivalent linear spring constant of one end cantilever frame section.

Then, for the first mode (translational)

$$M\ddot{x} + 2Kx = 0$$

$$\omega_1^2 = \frac{2K}{M}$$

for the second mode (torsional) Fig. 5

$$I\ddot{\varphi} + 2Ka^2\varphi = 0$$

$$\omega_2^{\,2} = \frac{2Ka^2}{I}$$

also: $$I = \frac{1}{12} M (4a^2 + b^2) \quad \text{for a uniform slab.}$$

so: $$\frac{\omega_2^{\,2}}{\omega_1^{\,2}} = \frac{12a^2}{4a^2 + b^2} \quad .$$

For the building under test, $2a = 65$ ft., $b = 37$ ft.,

$$\frac{\omega_2}{\omega_1} = 1.51 \quad .$$

A typical oscillograph record showing the torsional mode resonance condition is shown in Fig. 10, and Fig. 11 is the resonance curve plotted from this record. From the oscillograms the frequency of the first mode was found to be 3.62 cps, and that of the second mode 5.23 cps. This gives a ratio of 1.45, which is believed to be as close to the theoretical 1.51 as could be expected, since the actual floor slab may depart somewhat from the simple form assumed in the analysis,* and since the accuracy of the first mode is doubtful because of the low record amplitudes obtained, which were of the order of $\left(\frac{\omega_1}{\omega_2}\right)^4 = 0.19$ times the amplitudes of the second mode records of Fig. 10. In fact, it is believed that the theoretically determined frequency ratio when applied to the more accurately determined second mode frequency will give a better value for the lowest frequency than can be measured from the record, hence we conclude:

$$\omega_1 = \frac{5.23}{1.51} \approx 3.5 \text{ cps} \quad .$$

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* The slab actually had a rectangular cut out near the center and two pieces of equipment were supported near the ends of the slab. The weight of the equipment was very nearly equal to the weight of concrete absent from the hole, so that the total mass was probably very close to that of a uniform slab but the moment of inertia was perhaps somewhat larger.

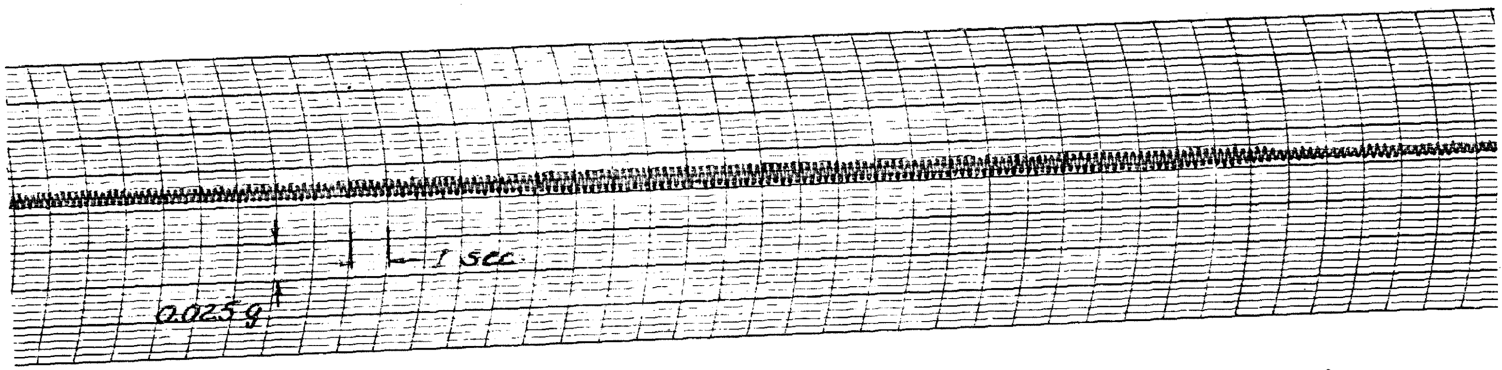

FIG. 10. ACCELERATION - TIME RECORD. TORSIONAL MODE
MILLER ACCELEROMETER - BRUSH CARRIER SYSTEM & RECORDER

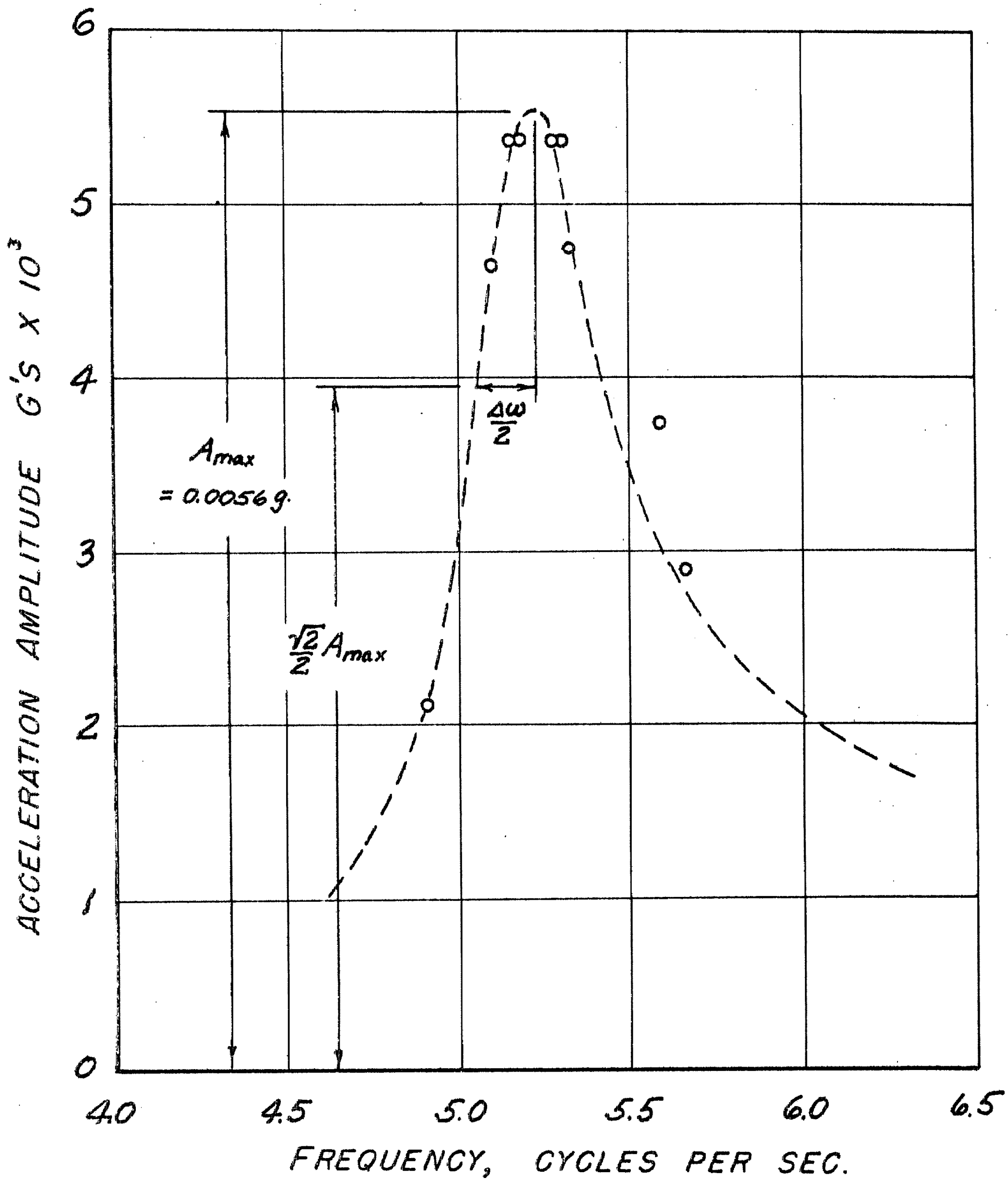

FIG. 11. RESONANCE CURVE
TORSIONAL MODE

Determination of Damping

The resonance curve of Fig. 11 is sufficiently well defined so that a value of the equivalent viscous damping can be determined by measuring the width of the resonance curve. Since the lower frequencies can be more accurately measured from the record than the higher ones, the rising portion of the resonance curve was used for this determination. If $\Delta\omega$ is the width of the resonance curve at an amplitude $\sqrt{2}/2$ times the resonant amplitude, it can be shown for small damping of the magnitude existing in the present test that the fraction of critical damping is $n = \frac{1}{2}\frac{\Delta\omega}{\omega}$, where ω is the resonant frequency. For the resonance curve of Fig. 11 this gives a value $n = 0.034$.

The dotted curve of Fig. 11 is a computed acceleration resonance curve for a single-degree-of-freedom system, plotted from the equation:

$$\text{acceleration} = \frac{K'\left(\frac{\omega}{\omega_n}\right)^4}{\sqrt{\left[1-\left(\frac{\omega}{\omega_n}\right)^2\right]^2+\left[2n\left(\frac{\omega}{\omega_n}\right)\right]^2}}$$

where the two unknowns K' and ω_n are determined from the experimentally obtained damping and resonant frequency. It will be seen that the system behaves essentially like a single-degree-of-freedom system as far as this mode of oscillation is concerned.

By referring to Fig. 5 it will be seen that the same structural members and the same structural deformations are involved in both the translational and the torsional modes. The only difference is that in the translational mode the two end frame structures move in phase, while in the torsional mode they are 180° out of phase. The same value of damping as was experimentally determined for the torsional mode would thus be expected to apply as well to the translational mode.

The low value of the damping as determined from the present test (3.4%) shows that large dynamic amplification effects are possible in structures of this type. For this value of damping the maximum dynamic amplification factor is about 15. This value of 3.4% damping may be compared with a value of 7% obtained for a reinforced concrete building,[2)] and with values of around 3% which apply to typical aircraft structures.

Comparison of Transient and Steady State Tests

In Fig. 12 is shown the ground acceleration spectrum for the quarry blast, obtained from an electric analog spectrum analysis of the ground acceleration measurements. The maximum recorded acceleration at the upper story position, as given in the preceding report, was 0.11 g. While it was not possible to get a satisfactory value of the structural damping from the transient tests, the spectrum shows that small variations of damping have only a small influence on response in the frequency range under consideration. From Fig. 12 it will be seen that the period corresponding to the maximum observed acceleration of 0.11 g is approximately 0.34 sec., or a frequency of 2.9 cycles per second. This figure should be compared with the translational frequency determined from the steady state shaking tests, which was 3.5 cycles per second. The difference between these two determinations seems definitely to be larger than can be explained by experimental errors alone, and we shall now consider possible differences in the two tests which might influence the results.

One of the significant differences between the transient and the steady state tests was the amplitude level of the maximum accelerations. While accelerations of higher than 0.1 g were involved in the transient tests, the accelerations in the steady state test were of the order of 0.005 g. This difference in acceleration amplitude level could have two distinctly different effects on the results. In the first place the damping in the structure could be considerably greater at the higher amplitudes, and it will be seen from the spectrum of Fig. 12 that this would increase the frequency as determined from the transient test. From these spectrum curves it will be found that if the damping had been from 15 - 20% of critical damping there would be substantial agreement in the frequencies as determined from the two tests. While this value seems larger than would be

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2) Alford, J. L., and Housner, G. W., A Dynamic Test of a Four-Story Reinforced Concrete Building, Bulletin of Seismological Society of America, Vol. 43, No. 1, Jan. 1953.

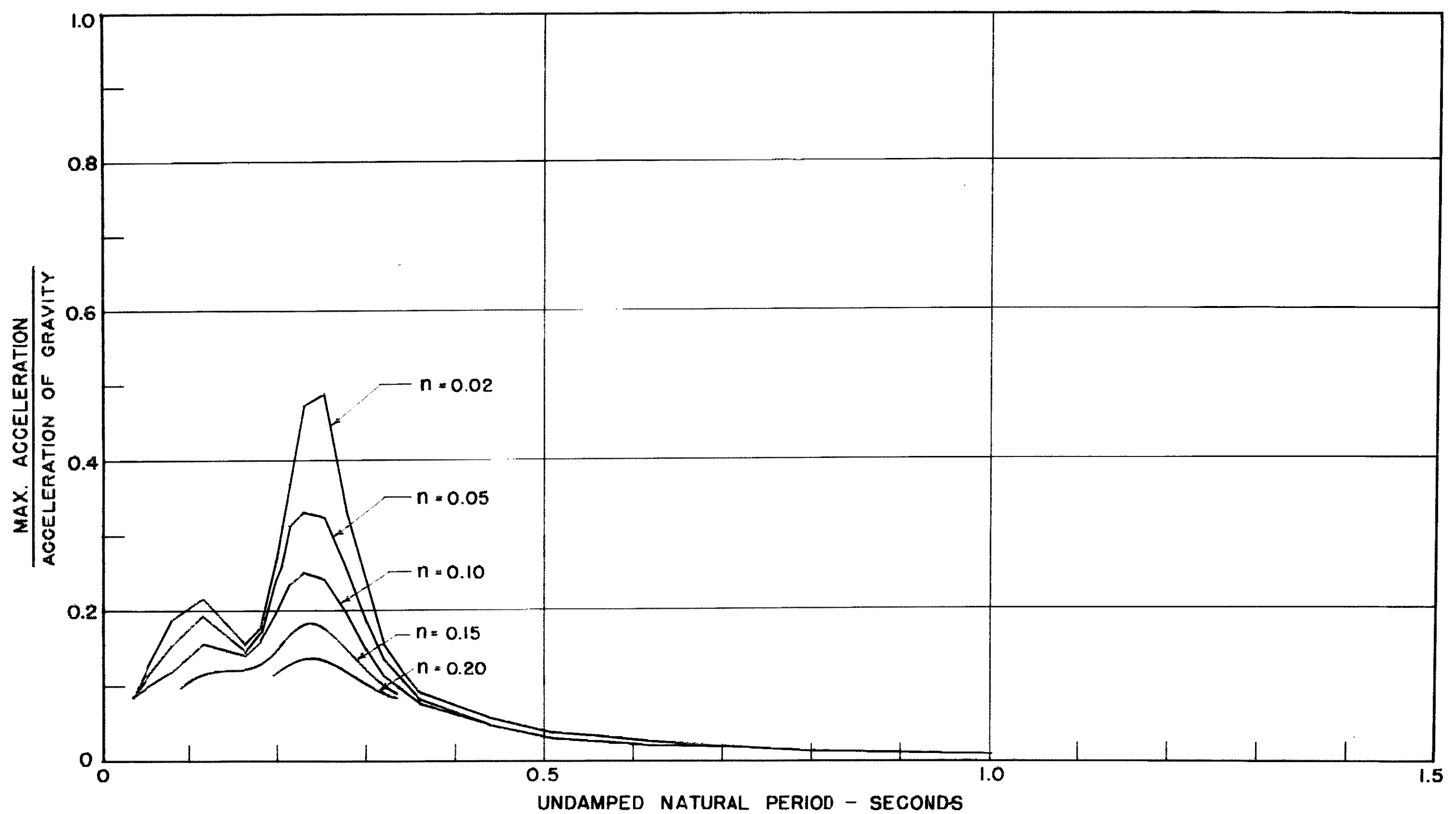


FIG. 12 ACCELERATION SPECTRA FOR CORONA QUARRY BLAST, JULY 26, 1952. E - W

expected, it should be remembered that the transient test subjected the building to unusually large motions, and that relative displacements between structural elements might account for the required energy dissipation. Unfortunately the data at hand does not permit any more conclusive decision as to the actual amount of damping present during the transient test.

A second effect caused by the large motions might be the introduction of a pronounced non-linear restoring force. If the structure were loaded to the point where some plastic yielding, or buckling of compression members, were to occur, then the non-linear character of the restoring force would result in an effective lower spring constant which would give the lower frequency required for agreement. With this thought in mind, calculations were made of the forces acting in the structure and of the stresses in the compression members. It was found that even under the large inertia forces of the blast motion, the axial compression forces did not approach the buckling value, and that no significant structural members were stressed into a markedly plastic region. It thus appears that non-linearities of this type are not responsible for the major differences between the two tests.

Another possible source of difficulty in the tests now requires consideration. In the calculations for the transient test it was assumed that the structure could be approximated by a one-degree-of-freedom system. Actually, of course, the structure is much more complicated than this, and one of the complications is such that the response of the building to the ground motion of the blast can be quite different than its response to the mechanical oscillator, which applies a force at just one point in the structure.

In Fig. 13 may be seen the essential structural features of the whole test building. It was the tall end of the structure, shown in detail in Fig. 3, which in the preceding portion of the report has been considered as a separate structure. We must now consider the influence that the lower portion of the building might have on the tests.

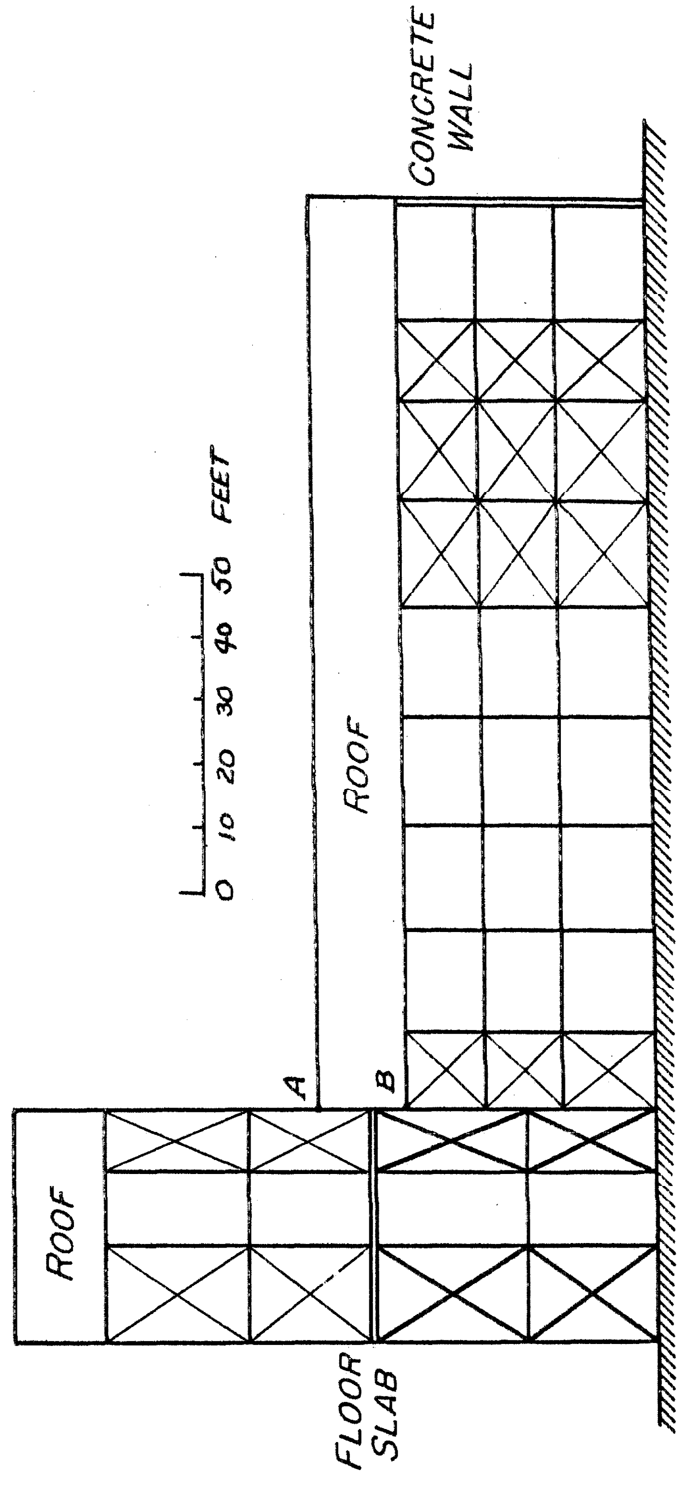


FIG. 13. BASIC STRUCTURE OF TEST BUILDING

It will be seen from Fig. 13 that the lower portion of the building connects into the upper portion at intermediate points in the spans of the vertical members as at A and B. Because of bending of these vertical members there is a relatively soft spring coupling between the portions, and it was this fact which originally suggested that the higher portion of the structure could be treated as a separate unit. A more complete dynamic model of the whole structure is the two-degree-of-freedom system shown in Fig. 14. M_1, K_1 represents the higher portion, M_2, K_2 the lower portion, and the spring K_3, which is considerably less stiff than either K_1 or K_2, represents the coupling between the two parts of the structure.

An examination of the two-degree-of-freedom system of Fig. 14 will show that its behavior would be expected to be somewhat different in the two tests. Fig. 15a shows the loading condition in the mechanical oscillator test, and Fig. 15b shows the loading condition during the quarry blast ground motion. One would expect that the loading of Fig. 15a would excite primarily the higher mode of the system, which should have a frequency very close to the separate frequency of M_1, K_1 alone, since K_3 is small. On the other hand, the loading of Fig. 15b would be expected to excite primarily the lower mode of vibration of the system, for which both masses would be moving in the same direction. That the mechanical oscillator test determined essentially the M_1, K_1 frequency is also likely from the fact that this frequency was determined from the torsional mode of the higher portion of the building, which would not be as much affected by the coupling of the lower portion of the building as the translational mode would be.

In an effort to get a quantitative check on the above ideas, calculations of the masses and stiffnesses of the two portions of the building were made. As nearly as could be determined, the masses and stiffnesses were within ± 15% of the following values:

$$M_1 = 3800 \text{ lb sec}^2/\text{ft.}$$
$$M_2 = 4000 \text{ lb sec}^2/\text{ft.}$$
$$K_1 = 150,000 \text{ lb/in.}$$

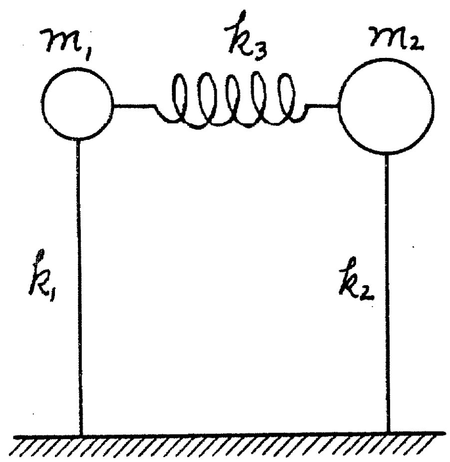
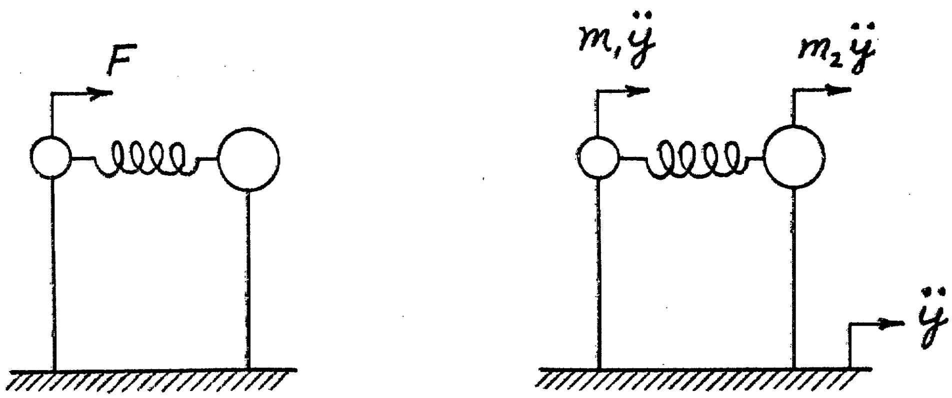


FIG. 14. TWO-DEGREE-OF-FREEDOM DYNAMIC MODEL



(a) MECHANICAL OSCILLATOR TEST

(b) GROUND MOTION BLAST TEST

FIG. 15. LOADING CONDITIONS DURING STEADY-STATE AND TRANSIENT TESTS

$$K_2 = 100,000 \text{ lb/in.}$$
$$K_3 = 10,000 \text{ lb/in.}$$

The two frequencies corresponding to the two modes of this system are:

$$\omega_1 = 2.88 \text{ cps} , \quad \omega_2 = 3.59 \text{ cps} .$$

The generalized forces corresponding to the two modes are approximately equal for the ground motion excitation of Fig. 15b, while for the mechanical oscillator test of Fig. 15a the generalized force for the higher frequency mode is about six times that for the lower. This tends to confirm the notion that the oscillator test excited mainly the motion of the higher portion of the building while the ground motion produced by the blast would excite also the lower mode involving the whole structure. One might expect that the ground motion would excite both modes simultaneously, and that both frequencies should appear in the accelerogram of the building motion. The size of the record obtained during the blast and the compressed time scale used made it difficult to resolve the shape of the accelerogram into such detail, and all that can be said is that the tests do not disprove this possibility. It will be noticed that the spread between the two calculated frequencies for the two modes agrees very well with the values experimentally obtained during the transient and the steady state tests. In view of the uncertainties as to the values of the effective masses and spring constants involved, the rather close numerical agreement of the above figures is perhaps fortuitous. However, it appears that the mechanism described could easily account for the differences in the test results under the transient and the steady state conditions.

Since many steady state oscillator tests have been made on buildings with the idea of determining building response to earthquakes, it is well to emphasize the fact that the earthquake ground motion may excite modes and frequencies in quite a different way than a force applied at a single point in a structure. This is a problem which has given much trouble in similar vibration tests of aircraft structures. It has been found that in order to properly excite the various wing and fuselage modes a number of vibration exciters located at various points throughout the structure, all operated simultaneously with the proper phase relationships, are necessary.[3)]

In view of the above mentioned differences between the transient test and the steady state test, it is concluded that the agreement between them is satisfactory, and that the general methods used for both tests are capable of sufficient accuracy for engineering applications.

Acknowledgements

We should like to thank the Minnesota Mining and Manufacturing Company, particularly Mr. R. E. Gundlach, Regional Manager, and Mr. G. J. LaVenture, Production Superintendent of the Corona Quarry, for permission to make the tests and for their very helpful cooperation. Thanks are also due to Mr. W. K. Cloud of the United States Coast and Geodetic Survey for making arrangements for the use of the vibration exciter.

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3) Lewis, R. C., and Wrisley, D. L., "A System for the Excitation of Pure Natural Modes of Complex Structure", Jour. Aero. Sci., Nov., 1950.